AIAA 82-4112

# Aircraft Pitch Attitude as a Performance Parameter

M.E. Eshelby*
*Cranfield Institute of Technology, Cranfield, Bedfordshire, England*

## Nomenclature

$C_D$ = aircraft drag coefficient
$C_{D_Z}$ = zero-lift drag coefficient
$C_L$ = aircraft lift coefficient
$C_{L_\alpha}$ = aircraft lift-curve slope
$E_{max}$ = aerodynamic efficiency $(L/D)_{max}$
$F_N$ = net thrust
$K$ = lift-dependent drag factor
$u$ = dimensionless speed, $V/V_{\overline{md}}$
$\alpha$ = wing incidence, measured from zero lift
$\alpha_0$ = wing zero-lift incidence
$\gamma$ = flight path gradient, climb positive
$\theta_b$ = pitch attitude, aircraft body axis
$\theta_{wb}$ = wing-body setting angle
$\theta_z$ $=\theta_b+\theta_{wb}+\alpha_0$
$\tau$ = dimensionless thrust, $F_N E_{max}/mg_0$

## Introduction

THE pitch attitude of an aircraft during flight should be considered at the design stage since in transport operations the pitch attitude of the cabin may be contributory to the safety and comfort of the passengers and crew. If the angle of the cabin floor to the horizontal is steep, it will present a hazard to passengers and cabin staff moving about the aircraft. There may also be a psychological effect on some passengers, particularly if there is no external horizontal reference visible, which may lead them to disorientation. It is preferable to maintain a near-level cabin pitch attitude throughout the flight in as much as possible; attitude changes of large magnitude are inevitable during takeoff and landing but, since all passengers are seated and secured with seat belts and the cabin cleared of moveable articles, this presents no particular hazard. Once the enroute climb is initiated it should be possible to restrict the aircraft body axis system (assumed to be parallel to cabin floor) to very small limits, say between ±2 deg with respect to the horizontal, throughout the flight until the later stages of the descent. This can be achieved by using the changes in zero-lift incidence with flap setting, and associated changes in zero-lift drag, to determine the relationship between aircraft body attitude and wing incidence and equating the difference to flight path gradient.

## Analysis of Attitude-Directed Performance

In the case of a subsonic aircraft operating below its critical Mach number, the drag can be expressed in the familiar parabolic form

$$C_D=C_{D_Z}+KC_L^2$$

where $C_{D_Z}$ and $K$ are constants.

Received July 9, 1981; revision received Nov. 17, 1981. 

*Lecturer in Applied Mechanics of Flight, College of Aeronautics. Member AIAA.

The incidence of the aircraft $\alpha$ can be determined from the pitch attitude of the aircraft body axis, the wing-body setting angle (which is usually constant), the wing zero-lift incidence, and the flight path gradient, thus

$$\alpha=(\theta_b+\theta_{wb}+\alpha_0)-\gamma=\theta_z-\gamma$$

and the dimensionless speed of the aircraft can then be written in terms of the dimensionless thrust $\tau$,[1]

$$u=\left\{(\tau-\theta_Z E_{max})+\left[(\tau-\theta_Z E_{max})^2-\left(1-\frac{1}{KC_{L_\alpha}}\right)\right]^{1/2}\right\}^{1/2}$$

Since the configuration of the aircraft in terms of the flap setting will determine the values of $K$, $\alpha_0$, $E_{max}$, and $C_{L_\alpha}$ it can be seen that for a specified value of body attitude the dimensionless speed and flight path gradient are uniquely defined by dimensionless thrust.

## Sample Application to Transport Aircraft

Although any use of pitch attitude as a performance parameter must be specific to a particular aircraft, the principle can be demonstrated by application to an aircraft defined only by basic parameters. Table 1 summarizes typical data for a transport aircraft. It is assumed that the lift-curve slope is not changed by flap deflection, although this in no way precludes such changes, and that the main effects of flap deflection are the variation in wing zero-lift incidence and aircraft drag characteristic. The lift incidence relationship is given in Table 1 for four flap settings typical of this class of aircraft.

Figure 1 shows the flight path gradient and rate of climb or descent for the four flap settings detailed in Table 1. These have been calculated for body attitude horizontal, 2 deg nose up, and 2 deg nose down. Since the aircraft weight is unspecified the thrust remains in a dimensionless form and also since the altitude and temperature are unspecified, the rate of climb or descent is given in equivalent terms as $\sigma^{1/2}(dH/dt)$.

From Fig. 1 it can be seen that the change in zero-lift incidence due to flaps can be used to produce a level flight attitude under most flight conditions. In the climb it is not generally possible to match the best rate or gradient of climb performance with level attitude since the flap deflection required to produce the large nose-down zero-lift incidence would be too great and the penalty incurred by the increase in drag would outweigh any advantage gained. In a high-speed

**Table 1 Aircraft data**

Wing loading, $mg_0/S$ = 4800 N/m² (100.25 lb/ft²)
Lift-curve slope, $C_{L_\alpha}$ = 5.5/rad (0.096/deg)
Wing-body angle, $\theta_{wb}$ = +2 deg

| Configuration | Drag characteristic | $\alpha_0$, deg | $E_{max}$ | $V_{e_{\overline{md}}}$, knots | $C_{L_{\overline{md}}}$ |
|---|---|---|---|---|---|
| 1) Cruise, flaps 0 deg | $0.02+0.05C_L^2$ | +2 | 15.81 | 216.1 | 0.632 |
| 2) Climb, long range descent, flaps 12 deg | $0.025+0.052C_L^2$ | +4 | 13.87 | 206.4 | 0.693 |
| 3) High-rate descent, flaps 30 deg | $0.05+0.057C_L^2$ | +7 | 9.37 | 177.6 | 0.937 |
| 4) High-gradient descent, flaps 50 deg + U/C down | $0.10+0.065C_L^2$ | +10 | 6.20 | 154.3 | 1.240 |

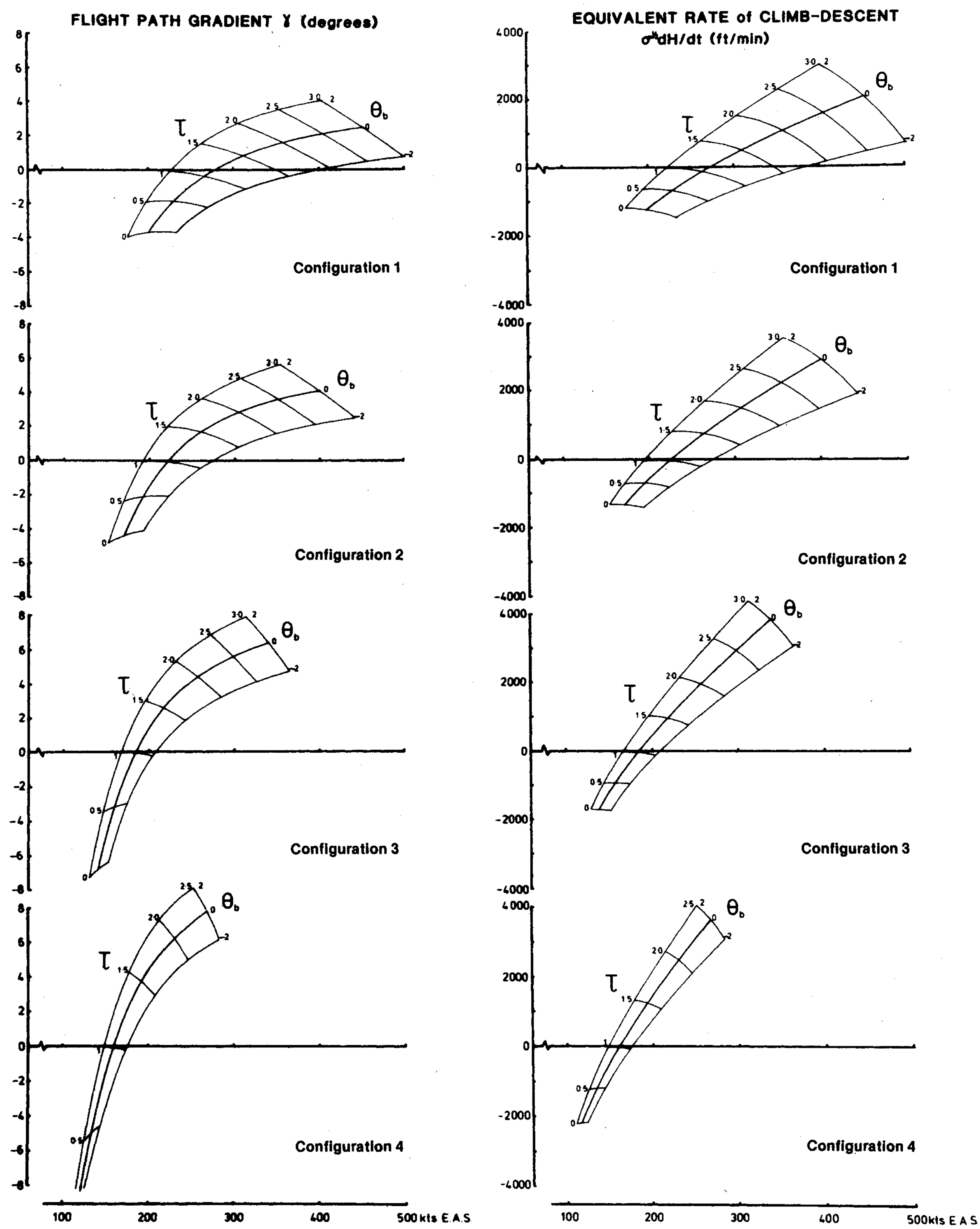


Fig. 1 Calculated flight path.

climb, using a clean configuration or small flap deflection, it is possible to produce a climb only at very low nose-up attitudes or in a level attitude by using airspeed and thrust in excess of that required for the level attitude cruise. The level attitude cruise can be matched closely to the optimum cruise speed by choice of the wing setting angle with respect to the body axis at the design stage.

In the descent flap deflection can be used to advantage to reduce the nose-down attitude of the aircraft. The decreased zero-lift incidence can be employed to offset the flight path angle so that a near-level attitude can be maintained over a substantial range of descent paths. Also high-drag devices, air brakes, or undercarriage can be employed to increase the lift-independent drag and thus reduce the minimum drag speed. This can be used to produce a flight path close to the optimum for best rate or gradient descents or for optimum speed descents.

In this illustrative analysis no account has been taken of any practical limits, stall speed, equivalent airspeed, or Mach limits or maximum thrust, since these would be peculiar to the aircraft concerned. Such limits could easily be applied to the calculated performance. Also the effect of flap has been considered only at set positions; if the aircraft has infinitely variable flap setting, then use of flap and attitude for descent control is possible. There are further possibilities of incorporating such control in a CCV mode.

### Reference

[1] Miele, A., *Flight Mechanics*, Vol. 1: "Theory of Flight Paths," Pergamon Press, New York, 1962, Chap. 9, pp. 149-189.

---

AIAA 82-4113

# Added Mass and the Dynamic Stability of Parachutes

John A. Eaton*
*Edgley Aircraft Ltd., Cambridge, England*

## Introduction

OVER the last two decades advances in the modeling of parachute dynamics and stability have been concentrated in the simulation of the solid body components of systems with ever-increasing numbers of degrees of freedom, made possible by the availability of ever-more powerful computers. In contrast, the fluid components of parachute systems have long languished for want of attention. One of the basic fluid components is the subject of this Note, which reviews the implementation of the added fluid mass in stability studies of parachutes. The complete form of the added mass tensor for a rigid axisymmetric parachute is obtained and implemented correctly for the first time, and results[1] indicate that added mass effects are even more significant than hitherto predicted.[2-5] The equations of motion in the Tory and Ayres model[5] are shown to be incorrect for the assumptions adopted.

## Added Mass in the Equations of Motion

Consider a dynamical system consisting of an arbitrary body moving in an arbitrary (real or ideal) fluid. Choose an orthogonal Cartesian body axis set $Oxyz$ and let $P_i$, $H_i$, $(i=1,2,3)$ be the component linear momenta and the component angular momenta of the system referred to $Oxyz$. The origin inertial velocity and angular velocity components are $V_i$ and $\omega_i$, and the external forces and moments acting on the system are $F_i$ and $M_i$. The general form of the Euler equations of motion for the complete system may then be written as

$$F_i = \dot{P}_i + \epsilon_{ijk}\omega_j P_k \tag{1}$$

$$M_i = \dot{H}_i + \epsilon_{ijk} V_j P_k + \epsilon_{ijk}\omega_j H_k \tag{2}$$

Using Hamilton's principle, Kirchhoff[6] showed in detail how, for motion of a rigid body in an ideal fluid, the external forces and moments on the body due to the fluid inertia may be derived from the added mass tensor and the generalized velocities of the body coordinate origin. Analogous to the solid body inertia tensor ($[B]$[7]), principal axes, and mass center, there exist an added inertia tensor ($[A]$), principal axes, and a center of added mass. Referring to Ibrahim,[7] we find the equivalent external forces and moments exerted on the fluid [Ref. 7, Eqs. (3.46) and (3.47)] are

$$F_{F_i} = A_{i\alpha}\dot{V}_\alpha + \epsilon_{ijk}\omega_j A_{k\alpha} V_\alpha \tag{3}$$

$$M_{F_i} = A_{i+3,\alpha}\dot{V}_\alpha + \epsilon_{ijk} V_j A_{k\alpha} V_\alpha + \epsilon_{ijk}\omega_j A_{k+3,\alpha} V_\alpha \tag{4}$$

where

$$A_{i\alpha}\dot{V}_\alpha = A_{ij}\dot{V}_j + A_{i,j+3}\dot{\omega}_j \qquad (\alpha = 1,\ldots,6;\ i,j=1,2,3)$$

$$A_{k\alpha}V_\alpha = A_{kl}V_l + A_{k,l+3}\omega_l \qquad (k,l=1,2,3)$$

If $B_{ij}$ is of dynamic significance, the corresponding $A_{ij}$ will be significant when its inertia or mass ratio $\mu_{ij} = A_{ij}/B_{ij} \geq O(1)$.

## Previous Representations of the Added Mass Tensor

For the basic stability modeling of a parachute with a fully deployed canopy, conventional practice has been to assume the canopy to be rigid and axisymmetric and the fluid to be inviscid, irrotational, and incompressible.

Henn[8] was one of the first to take added mass into account in a planar, three degree-of-freedom study of parachute stability. Replacing the canopy with a rigid ellipsoidal, air-filled body, his added mass components comprised contributions from the fluid regions interior ("included mass" and "included moment of inertia") and exterior ("additional apparent" masses and moment of inertia) to the body. In the present notation (see next section) his added mass components were $A_{11}$, $A_{33}$, and $A_{55}$. By analytical solution of the three linearized equations of motion, Henn demonstrated strong influences on the damping and frequency of lateral oscillations (of a mass-geometrically typical personnel parachute) due to individual variations of $A_{11}$ and $A_{33}$ within the ranges $\mu_{11} = 0.0$-$0.6$ and $\mu_{33} = 0.0$-$1.0$ for baseline values of $\mu_{11} = \mu_{33} = 0.5$.

Henn's equations were widely used until 1962, when Lester[9] showed that they were erroneous; in uncritically applying to the added mass components the rigid-dynamical equations, rather than the fundamental Kirchhoff equations,[6] Henn had violated the concept of added mass.

Ludwig and Heins,[2] again for planar three degree-of-freedom motion, confined their added mass implementation to Henn's two isotropic "included" terms, but solved the nonlinear equations of motion using both analog and digital techniques. Using values of $\mu_{11}$ $(=\mu_{33})$ in the range of 0.6-1.4, they concluded that the added mass mainly affected oscillation amplitude and damping, had a minor effect on frequency, and overall was not very important.

White and Wolf,[3] using the same added mass components as Ludwig and Heins, solved the five degree-of-freedom equations of motion for a single rigid body system and showed that stability decreased with increasing $\mu$.

One exception to the trend of reducing the representation of added mass was the six degree-of-freedom rigid body model introduced in 1972 by Tory and Ayres,[5] where, on the basis of the dubious assumption of a real (physical) distinction between "included" and "apparent" mass,[10] they reverted to Henn's original collection of components. As part of the overall model validation[1] the author carried out a sensitivity analysis[4] for a typical personnel parachute system. This indicated that $A_{33}$ and $A_{55}$ were not important in regard to stability, but that $A_{11}$ had a significant effect on damping. Damping increased with increasing store mass, as found by White and Wolf, but was unaffected by $A_{33}$. Baseline mass ratio values were used $\mu_{11}$ $(=\mu_{22}) = 0.5$, $\mu_{33} = 0.9$, and the ranges covered were $\mu_{11} = 0.0$-$6.7$ and $\mu_{33} = 0.0$-$6.5$.

---

Received Aug. 17, 1981; revision received Nov. 16, 1981. 

*Consulting Engineer. Member AIAA.